A column generation based approach for the joint order batching and picker routing problem

Olivier Briant[1], Hadrien Cambazard[1], Diego Cattaruzza[2], Nicolas Catusse[1], Anne-Laure Ladier[3], Maxime Ogier[2]

[1] Univ. Grenoble Alpes, CNRS, Grenoble INP, G-SCOP, 38000 Grenoble, France
{olivier.briant,hadrien.cambazard,nicolas.catusse}@grenoble-inp.fr
[2] Univ. Lille, CNRS, Centrale Lille, Inria, UMR 9189 - CRIStAL, F-59000 Lille, France
{diego.cattaruzza,maxime.ogier}@centralelille.fr
[3] Univ Lyon, INSA Lyon, DISP laboratory EA 4570, F-69100 Villeurbanne, France
anne-laure.ladier@insa-lyon.fr

Mots-clés : *order batching, picker routing, column generation.*

1 Introduction

Picking is the process of retrieving products from the inventory and is often considered a very expensive operation in warehouse management. A set of pickers perform routes into the warehouse, pushing a trolley and collecting items to prepare customer orders. However, customer orders usually do not fit the capacity of a trolley. They are typically grouped into batches, or on the contrary divided into subsets, with the aim of collecting all the orders by minimizing the walked distance. This problem is known as the joint order batching and picker routing problem [1].

This work presents an exponential linear programming formulation where variables or columns are related to single picking routes in the warehouse. More precisely, a column refers to a route involving a set of picking operations and satisfying the side constraints required at the trolley level such as the mixing of orders or the capacity. Computing such a picking route is an intractable routing problem in general and, depending on the warehouse layout, can closely relate to the traveling salesman problem (TSP). The rational of our approach is however to consider that the picking problem alone, in real-life warehouses, is easy enough in practice to be solved exactly. We apply this approach on two different industrial benchmarks, based on different warehouse layouts.

2 Problem specification and industrial applications

The warehouse layout is modeled as a directed graph $G = (\mathcal{V}, \mathcal{A})$ with two types of vertices, locations and intersections. Locations contain one or more product references to be picked.

Two typical examples of warehouse layouts are used as benchmarks in the present work.

— A regular rectangular layout made of vertical *aisles* and horizontal *cross-aisles*. Such a layout has been used by numerous authors in the past [2] to define the order picking problem. It is the setup of the Walmart benchmark.

— An acyclic layout where pickers are not allowed to backtrack. It is another typical industrial setup where the flow is constrained in a single direction and an aisle must be entered and exited on the same side. It is the setup of the HappyChic benchmark.

Each *product reference* $p \in \mathcal{P}$ is characterized by its location in the warehouse and its size V_p^w in each dimension $w \in \mathcal{W}$. A product reference may have several dimensions such as weight and volume and we refer to the set of dimensions as $\mathcal{W}$.

An order from a customer is defined as a set of order lines. An *order line* $l \in \mathcal{L}$ is related to an order o and defined as a pair (p_l, Q_l) where $p_l \in \mathcal{P}$ is a product reference and Q_l is the number of items to pick. An *order* $o \in \mathcal{O}$ is a set of order lines $\mathcal{L}_o \subseteq \mathcal{L}$. Moreover, an order can be split in at most M_o boxes.

Order lines are collected by trolleys, each carrying a set of $\mathcal{B}$ boxes. A box has a capacity V^w in dimension $w \in \mathcal{W}$ and an order line can be assigned into several boxes (the quantity Q_l of an order line l can be split among several boxes). A box is therefore filled with *partial order lines*. A partial order line $\tilde{l}$ is a pair $(p_l, \tilde{Q}_l)$ with $\tilde{Q}_l \leq Q_l$. A box can only contain partial order lines from a single order.

A solution is a collection of routes $\mathcal{R}$ in the warehouse layout G. Each route r is travelled by a trolley which collects partial order lines into its boxes. The capacities of the boxes must be satisfied in each dimension $w \in \mathcal{W}$. An order $o \in \mathcal{O}$ can not be assigned to more than M_o boxes. Finally all order lines must be picked with the required number of items. The objective is to minimize the total distance to perform all the routes in $\mathcal{R}$.

The two industrial cases addressed in the present work, from the Walmart and HappyChic, differ slightly. In particular, for the Walmart case, only one dimension is considered for a box, representing the maximum number of items in a box. Additionally, an order must be picked entirely by a single trolley.

3 A column generation based approach

In the industrial case of HappyChic, the picking takes place on an acyclic graph, thus boils down to an easy path problem. In Walmart's case, warehouse have the regular rectangular structure made of aisles and cross-aisles. In that case, dynamic programming algorithms can take advantage of that structure to efficiently solve the corresponding TSP when the warehouse contains up to eight cross-aisles, which is beyond most real-life warehouse's sizes [3]. We therefore assume that in both cases, an efficient oracle is available to provide optimal picking routes in the warehouse.

We show that such an oracle allows for a very effective exponential LP formulation of the joint order batching and picking problem. The pricing problem can be seen as a prize-collecting TSP with a capacity constraint and the pricing algorithm heavily relies on the picking oracle to generate cutting planes. A number of improvements are proposed to speed up the pricing. In particular, a procedure to strengthen the cutting planes is given when the distance function for the considered set of orders is submodular. For the industrial case of HappyChic, the graph is acyclic, so it is possible to propose a polynomial set of constraints to exactly calculate the distance, instead of generating cutting planes.

The proposed formulation is compared experimentally on Walmart's benchmark and proves to be very effective, improving many of the best known solutions and providing very strong lower bounds. Finally, this approach is also applied to the HappyChic case, demonstrating its generality and interest for this application's domain.

Références

[1] Cristiano Arbex Valle, John E. Beasle, and Alexandre Salles da Cunha. Optimally solving the joint order batching and picker routing problem. *European Journal of Operational Research*, 262(3) :817–834, 2017.

[2] René De Koster, Tho Le-Duc, and Keed Jan Roodbergen. Design and control of warehouse order picking : A literature review. *European Journal of Operational Research*, 182(2) : 481–501, 2007.

[3] Hadrien Cambazard, and Nicolas Catusse. Fixed-Parameter Algorithms for Rectilinear Steiner tree and Rectilinear Traveling Salesman Problem in the plane. arXiv preprint arXiv :1512.06649, 2015.